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基于欧拉方程的二维振荡机翼非定常气动设计反命题方法

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摘 要:建立了基于欧拉方程组的二维振荡机翼非定常气动设计反命题方法的数学模型。通过一系列变换,将物理时空中的求解域转换成映象坐标系中的规范区域;并导出了映象坐标下的欧拉方程,结合非定常反命题的边界条件,便可用有限差分方法求解。本方法引入映象坐标系解决反命题边界形状的不确定性,并能利用欧拉方程现有的各种差分格式于反命题求解。

关键词:非定常反问题;欧拉方程;振荡机翼;映象空间
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0 引 言

尽管人们已对包括外流与内流、理想与粘性流体在内的各种非定常气动正问题进行过大量研究与计算分析[1,2],但对非定常气动设计反问题的研究目前还刚开始。文献[3]首次深入探讨这类问题的适当提法,建立了二维机翼非定常反问题的变域变分原理;并且在文献[4]中将之发展为多工况反命题的解法。这些解法都是以有势流动为基础,求解域定义在物理时空中,并须借助变域变分方法来处理反问题的不确定边界。本文用欧拉方程组代替势流方程作为解法的基础,并把文献[5]中求解定常流反命题的通用解法推广到(x, y, t)三维时空,从而建立一条求解非定常反问题的新途径。我们先通过一系列坐标变换,将非定常欧拉方程组转换到时空映象坐标系。在映象坐标系下,求解域成为一个规范的矩形区域。而位置随着时间变动、几何形状不确定的机翼经变换后,成为一段固定的水平直线,这样就避免了未知边界的困难。采用欧拉方程组摆脱了有势流动的限制,有利于将方法向含有激波的跨、超声速范围推广;同时,现有的许多求解欧拉方程十分有效的差分格式可以直接用于反问题求解,这将有力地促进非定常设计反问题研究的发展。

1 映象坐标下的非定常欧拉方程

1.1 基本方程

在物理坐标系(x, y, t)中,欧拉方程组为:

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=0 \tag{1}$$

其中:

$$U=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ E\end{bmatrix};F=\begin{bmatrix}\rho u\\ p+\rho u^2\\ \rho uv\\ u(E+p)\end{bmatrix};G=\begin{bmatrix}\rho v\\ \rho uv\\ p+\rho v^2\\ v(E+p)\end{bmatrix}$$

而

$$E=\frac{p}{k-1}+\frac{\rho}{2}(u^2+v^2) \tag{3}$$

1.2 y 方向的坐标变换

为了克服翼面形状未知的困难,我们将文献[4、5]建议的贴体坐标系ξ,η推广到三维空间ξ-η-τ中去,取变换式为:

$$\tau=t,\xi=x \tag{4a}$$

对翼上方区域:

$$\eta=\eta_w+\frac{K_0}{\delta y_U(t,x)}[y(t,x)-y_s(t,x)]$$

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对翼下方区域：

$$\eta = \eta_L + \frac{K_0}{\delta y_L(t,x)}[y(t,x) - y_L] \qquad (4b)'$$

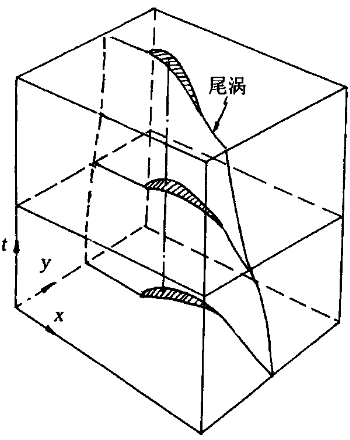


图 1　物理时空图

Fig.1　Physical coordinates

其中

$$\delta y_U = h_U - y_s(t,x)$$

$$\delta y_L = y_p(t,x) - h_L$$

诸符号意义见图 2(a)。为方便计，可选 $y_U = h_U, y_L = h_L = 0, \eta_L = 0, \eta_W = K_0, \ \eta_U = 2K_0$，于是(4b)'式变为：

$$\left.\begin{aligned} &\text{对翼上方域：}\\ &\eta = K_0 + \frac{K_0}{\delta y_U(t,x)}[y(t,x) - y_s(t,x)]\\ &\text{对翼下方区域：}\\ &\eta = \frac{K_0}{y_p(t,x)}y(t,x) \end{aligned}\right\} \quad (4b)$$

故以后的推导，可不分翼上、下方区域而写出统一形式，但在具体用于上、下方区域时，应以其对应的表达式代入。

图 3 显示映象坐标系与物理坐标系的关系。$\bar{\tau}$ 为在三维时空中一点的位置矢量，τ 是它在 $t-y$ 面上的投影矢量；α、β 分别是 $\bar{\tau}$ 在 $t-x$、$t-y$ 坐标面的投影与 t 轴的夹角，而 γ 则是 ξ' 轴的切向与 x' 轴的夹角。

由(4a)、(4b)式可写出：

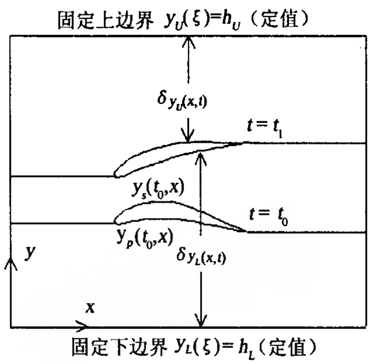
$$\left.\begin{aligned} dx &= d\xi,\\ dy &= \mathrm{tg}\gamma d\xi + H_\eta d\eta \end{aligned}\right\} \qquad (5)$$

且

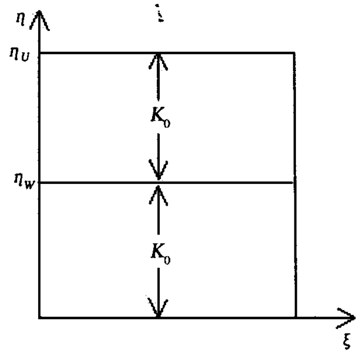
$$H_\eta = \frac{\delta y}{\partial \eta}(x) = \delta y/K_0 \qquad (6)$$

以 Φ 代表某一变量，则：

$$\frac{\partial \Phi}{\partial \xi} = \left.\frac{d\Phi}{dx}\right|_\xi = \frac{\partial \Phi}{\partial x}\frac{dx}{d\xi} + \frac{\partial \Phi}{\partial y}\frac{dy}{d\xi} = \frac{\partial \Phi}{\partial x} + \mathrm{tg}\gamma\frac{\partial \Phi}{\partial y}$$



(a)　物理平面



(b)　映象平面

图 2　y 方向的坐标变换

Fig.2　The coordinate transformation in y direction

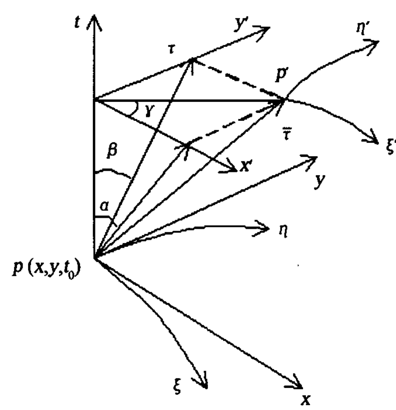


图 3　时空坐标系

Fig.3　Time-space coordinates system

$$\frac{\partial \Phi}{\partial \eta} = H_\eta \frac{\partial \Phi}{\partial y}$$

故而：

$$\left.\begin{aligned} \frac{\partial \Phi}{\partial x} &= \frac{\partial \Phi}{\partial \xi} - \frac{\mathrm{tg}\eta}{H_\eta}\frac{\partial \Phi}{\partial \eta}\\ \frac{\partial \Phi}{\partial y} &= \frac{1}{H_\eta}\frac{\partial \Phi}{\partial \eta} \end{aligned}\right\} \qquad (7)$$

又因

$$\frac{\partial \Phi}{\partial \tau} = \left.\frac{\mathrm{d}\Phi}{\mathrm{d}t}\right|_{\tau} = \frac{\partial \Phi}{\partial t} + \frac{\partial \Phi}{\partial x}\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\tau} + \frac{\partial \Phi}{\partial y}\left.\frac{\mathrm{d}y}{dt}\right|_{\tau}$$

$$= \frac{\partial \Phi}{\partial t} + \left(\frac{\partial \Phi}{\partial \xi} - \frac{\mathrm{tg}\gamma}{H_\eta}\frac{\partial \Phi}{\partial \eta}\right)\mathrm{tg}\alpha + \frac{\mathrm{tg}\beta}{H_\eta}\frac{\partial \Phi}{\partial \eta}$$

而且,由于取变换式为 $\xi = x$ 与 t 无关,使 $\mathrm{tg}\alpha = (\mathrm{d}x/\mathrm{d}t)|_\tau = 0$, 所以:

$$\frac{\partial \Phi}{\partial t} = \frac{\partial \Phi}{\partial \tau} - \mathrm{tg}\alpha\frac{\partial \Phi}{\partial \xi} + \left(\frac{\mathrm{tg}\alpha\mathrm{tg}\gamma - \mathrm{tg}\beta}{H_\eta}\right)\frac{\partial \Phi}{\partial \eta}$$

$$= \frac{\partial \Phi}{\partial \tau} - \frac{\mathrm{tg}\beta}{H_\eta}\frac{\partial \Phi}{\partial \eta} \tag{8}$$

将(7)、(8)代入(1),得到映象坐标(ξ,η,τ)下的欧拉方程:

$$\frac{\partial U}{\partial \tau} - \frac{\mathrm{tg}\beta}{H_\eta}\frac{\partial U}{\partial \eta} + \frac{\partial F}{\partial \xi} - \frac{\mathrm{tg}\gamma}{H_\eta}\frac{\partial F}{\partial \eta} + \frac{1}{H_\eta}\frac{\partial G}{\partial \eta} = 0 \tag{9}$$

1.3 ξ 方向的坐标变换

在假设 $\xi = x$ 的前提下,机翼摆动使前、后缘在 τ-ξ 面上的轨迹不是直线,如图 4 所示。若再做一变换,令 $\bar{\xi} = f(\xi,\tau)$ 可使前、后缘的轨迹在 $\bar{\tau}$-$\bar{\xi}$ 面上成为直线,现设 $f(\xi,\tau)$ 为:

$$\begin{cases} \xi/\xi_L & \xi < \xi_L \\ 1 + (\xi - \xi_L)/(\xi_T - \xi_L) & \xi_L < \xi < \xi_T \\ 2 + (\xi - \xi_T)/(L_\xi - \xi_T) & \xi > \xi_T \end{cases} \tag{10}$$

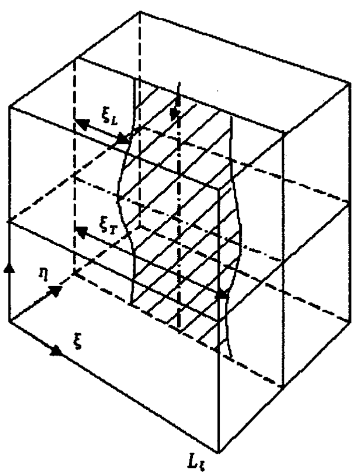


图 4 ξ-η-τ 坐标系

Fig.4 ξ-η-τ coordinates system

式中,前、后缘坐标 ξ_L 与 ξ_T 都是时间的函数,L_ξ 是求解域在 ξ 方向的总长度。这样,对 ξ、τ 的导数都要改成对 $\bar{\xi}$、$\bar{\tau}$的导数。

a) 上游区:

$$\frac{\partial}{\partial \xi} = \frac{1}{\xi_L}\frac{\partial}{\partial \bar{\xi}} \tag{11}$$

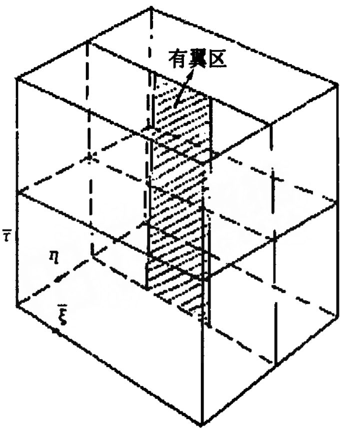


图 5 $\bar{\xi}$-η-$\bar{\tau}$ 坐标系

Fig.5 $\bar{\xi}$-η-$\bar{\tau}$ coordinates system

又因为:

$$\frac{\partial}{\partial \bar{\tau}} = \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \xi}\left.\frac{\mathrm{d}\xi}{\mathrm{d}\tau}\right|_{\bar{\tau}} = \frac{\partial}{\partial \tau} + \mathrm{tg}\alpha\frac{\partial}{\partial \xi}$$

所以:

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial \bar{\tau}} - \frac{\partial}{\partial \xi}\left.\frac{\mathrm{d}\xi}{\mathrm{d}\tau}\right|_{\bar{\tau}} = \frac{\partial}{\partial \bar{\tau}} - \mathrm{tg}\alpha\frac{\partial}{\partial \xi}$$

$$= \frac{\partial}{\partial \bar{\tau}} - \frac{\mathrm{tg}\alpha}{\xi_L}\frac{\partial}{\partial \bar{\xi}} \tag{12}$$

注:参阅图 6。

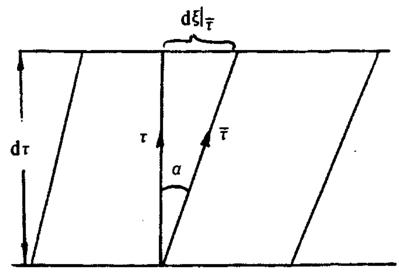


图 6 ξ 沿时间坐标的变化

Fig.6 The change of ξ along time coordinate

$$\mathrm{d}\xi|_{\bar{\tau}} = \xi_L\mathrm{d}\bar{\xi}|_{\bar{\tau}} + \bar{\xi}\,\mathrm{d}\xi_L = \bar{\xi}\,\mathrm{d}\xi_L \qquad (\mathrm{d}\bar{\xi}_{\bar{\tau}} = 0)$$

$$\mathrm{tg}\alpha = \left.\frac{\mathrm{d}\xi}{\mathrm{d}\tau}\right|_{\bar{\tau}} = \bar{\xi}\frac{\mathrm{d}\xi_L}{\mathrm{d}\tau} = \bar{\xi}(\mathrm{tg}\alpha)_L \tag{13}$$

$\bar{\xi}$其实只是个比例数,而

$$\mathrm{d}\xi_L = \xi_L(\tau + \mathrm{d}\tau) - \xi_L(\tau)$$

b) 翼片区:

$$\xi = (\bar{\xi} - 1)(\xi_T - \xi_L) + \xi_L$$

$$\frac{\partial}{\partial \xi} = \frac{1}{(\xi_T - \xi_L)} \frac{\partial}{\partial \bar{\xi}} \tag{14}$$

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial \bar{\tau}} - \frac{\partial}{\partial \xi}\frac{\mathrm{d}\xi}{\mathrm{d}\tau}\Big|_{\bar{\tau}} = \frac{\partial}{\partial \bar{\tau}} - \mathrm{tg}\frac{\partial}{\partial \xi}$$

$$= \frac{\partial}{\partial \bar{\tau}} - \frac{\mathrm{tg}\alpha}{(\xi_T - \xi_L)}\frac{\partial}{\partial \bar{\xi}} \tag{15}$$

$$d\xi \mid_{\bar{\tau}} = (\bar{\xi} - 1)\frac{\mathrm{d}(\xi_T - \xi_L)}{\mathrm{d}\tau}\mathrm{d}\tau + \frac{\mathrm{d}\xi_L}{\mathrm{d}\tau}\mathrm{d}\tau;$$

$$\mathrm{tg}\alpha = \frac{\mathrm{d}\xi}{\mathrm{d}\tau}\Big|_{\bar{\tau}} = (\bar{\xi} - 1)[(\mathrm{tg}\alpha)_T - (\mathrm{tg}\alpha)_L] + (\mathrm{tg}\alpha)_L \tag{16}$$

c) 下游区：

$$\xi = (\bar{\xi} - 2)(L_\xi - \xi_T) + \xi_T$$

$$\frac{\partial}{\partial \xi} = \frac{1}{(L_\xi - \xi_T)}\frac{\partial}{\partial \bar{\xi}} \tag{17}$$

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial \bar{\tau}} - \frac{\partial}{\partial \xi}\frac{\mathrm{d}\xi}{\mathrm{d}\tau}\Big|_{\bar{\tau}} = \frac{\partial}{\partial \bar{\tau}} - \mathrm{tg}\frac{\partial}{\partial \xi}$$

$$= \frac{\partial}{\partial \bar{\tau}} - \frac{\mathrm{tg}\alpha}{(L_\xi - \xi_T)}\frac{\partial}{\partial \bar{\xi}} \tag{18}$$

$$\mathrm{d}\xi \mid_{\bar{\tau}} = (3 - \bar{\xi})\frac{\mathrm{d}\xi_T}{\mathrm{d}\tau}\mathrm{d}\tau;$$

$$\mathrm{tg}\alpha = \frac{\mathrm{d}\xi}{\mathrm{d}\tau}\Big|_{\bar{\tau}} = (3 - \bar{\xi})(\mathrm{tg}\alpha)_T \tag{19}$$

2　边界条件

2.1　翼面边界条件

设翼片绕 O 轴以频率 ω 摆动，在 $t = 0$ 时刻翼面上一点(x_0, y_0)随时间的坐标变化为：

$$\left.\begin{aligned} x(x_0, t) &= r_0\cos(\delta + \theta_A\sin(\omega t)) \\ y(x_0, t) &= r_0 sin(\delta + \theta_A\sin(\omega t)) \end{aligned}\right\} \tag{20}$$

正问题边界条件：

$$v_n = -u\sin\gamma + v\cos\gamma$$

$$= y_0\theta_A\omega\cos(\omega t)\sin\gamma + x_0\theta_A\omega\cos(\omega t)\cos\gamma \tag{21}$$

反问题边界条件：

沿 $t - x_0$ 平面上一规定路线给出压力或速度分布条件：

$$\left.\begin{aligned} P &= P_{pr}(x_0) \\ \text{或}\qquad V &= V_{pr}(x_0) \end{aligned}\right\} \tag{22}$$

x_0 是指 $t = 0$ 时的 x 坐标。图 8 表示，在一个振荡周期中($0 \leqslant t \leqslant T$)，先沿着吸力面从前缘到后缘，再沿压力面从后缘到前缘规定速度随时间变化的目标曲线。这样在每一时刻，翼面可能有一个点处于反问题状态，其 y 坐标值是未知的，其余部分都作为正问题边界处理。然而，在 $t - x_0$ 平面上的两条规定路线也可以相交，那样在每一时刻，翼面就可能有两个点处于反问题状态，吸力、压力面各有一个。

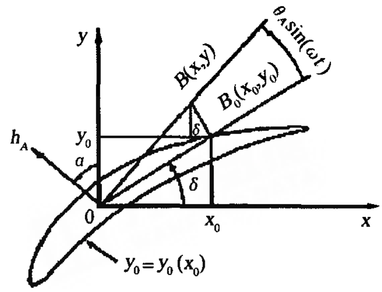


图 7　振荡机翼

Fig.7　Oscillating airfoil

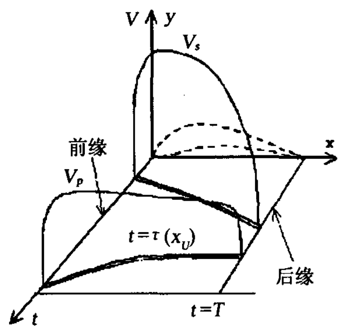


图 8　反问题条件

Fig.8　Condition of inverse problem

2.2　远场边界条件

采用无反射边界条件：

$$\rho V_n = \frac{\bar{\rho}}{\gamma - 1}(K_{\mathrm{I}} - K_{\mathrm{II}}) \tag{23}$$

其中 K_{I} 与 K_{II} 分别为沿左行与右行特征线的黎曼不变量。它们分别由上一时刻无穷远处与边界内侧的流动参数确定。$\bar{\rho}$ 为上一时刻的密度值。

3　结束语

本文首次利用欧拉方程组在映象坐标系中建立了非定常二维振荡机翼的反命题的数学模型。它完全摆脱了势流模型的局限，而且能提高非定常气动设计反问题的可解性。众所周知，机翼在振荡过程中，后缘不断有新的涡产生、并脱落进入尾流，形成尾涡面。尾涡面两侧压力相等，切向速度分量存在间断，它的位置和形状都随时间改变。这些复杂的因素与反问题固有的非线性结合在一起给数值求解造成极大困难。然而，非定常欧拉方程的差分解法有一系列

非常成功的离散格式,不但可用于跨、超声速计算时自动捕获激波,而且也可用于搜索跟踪尾涡面。以前,人们曾将欧拉方程组的正问题解法与定常反问题求解相结合构成试凑法[6],取得很好效果。现在把欧拉方程的离散格式引入非定常反问题的求解,预期可以减少其求解的难度,提高这类问题的可解性。

参 考 文 献:

[1] UZUN A, AKEY H U, BRONNENBERG C E. Parallel computations of unsteady Euler equations on dynamically deforming unstructured grids [A]. Proc. the parallel CFD '99' conf. [C]. VA USA: Williamsberg, 1999.

[2] ZHENYIN LI. Parallel computations of 3-D unsteady compressible Euler equations with structural coupling [R]. Master thesis, Dept. Mech. Engrg. Indiana Univ. USA, 2002.

[3] 刘高联. 二维机翼非定常气动力学反命题的变分理论 [J]. 空气动力学学报,1996, 14(1): 1-5.

[4] LIU G L. A general variational theory of multipoint inverse design of 2-D transonic cascades based on an artificial flow-oscillation model [J]. *Int. J. Turbo & Jet-Engines*, 1999, 16: 144-148.

[5] 刘高联. 旋成面叶栅一些杂交气动命题的新解法 [J]. 工程热物理学报,1984, 5(1):27-32.

[6] DEMULENAERE A. & Van den Braembussche, R.A. Three-dimensional inverse design method for turbine and compressor blades [A]. Symposium on Design Principles and Methods for Aircraft Gas Turbine Engines [C]. Toulouse, France, 1998.

An approach of inverse problem for unsteady aerodynamic design of 2-D oscillating airfoil based on Euler equations

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Abstract: A mathematical model for the inverse problem of unsteady aerodynamic design of 2-D oscillating airfoil based on Euler equations is proposed. The physical solution domain is transferred into a normalized domain in the image coordinates system through a series of transformation, so that the difficulties caused by the unknown geometry of airfoil can be avoided. The Euler equations are transformed into the image space, which, combined with the boundary conditions of the inverse unsteady problem, can be solved using the various FEM schemes now available.

Key words: inverse problem; unsteady flow; Euler equations; oscillating airfoil

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nozzle is studied by using V.N Uskov's method, i. e. differential dynamics compatibility condition, in which the calculating method of some parameters distribution behind shock wave along SW is given. By using this method, some results obtained are similar to the results of experiments. Meanwhile the positions of Mach disc are calculated for over-expansion jet. If triple configurations of stationary shocks are formed, the distributions of parameters relating to this structure are calculated and showed in the figuration. This method is able to be used to guide the parameters design for some gas dynamic equipments.

Key words: shock wave; conical Laval nozzle; parameter control